Barem simulare bacalaureat matematică, filiera teoretică, ştiinţele naturii, M2, 26.03.2013, GORJ.

SUBIECTUL I

1. $x=\frac{1}{\log_2 48}=\frac{1}{\log_2 2^4 3}=\frac{1}{4+\log_2 3}\Rightarrow \log_2 3=\frac{1-4x}{x}$ **(2pct);**

$\log_{108} 3=\frac{1}{\log_3 108}=\frac{1}{3+2\log_3 2}=\frac{1}{3+2\cdot\frac{1}{\log_2 3}}=\frac{1-4x}{3-10x}$ **(3pct).**

2. Relaţiile lui Viete scrise bine (2pct); calculul expresiei=4 (2pct); constatarea că este număr raţional (1pct).

3. $2n^2\le 3n+2$ **(1pct);** $n\in[-\frac{1}{2},2]\cap N=\{0,1,2\}$ **(1pct);** $n=0\Rightarrow C_2^0\ge 8$ **fals (1pct);**

$n=1\Rightarrow C_5^2\ge 8$ **adevărat, soluţie (1pct);** $n=2\Rightarrow C_8^8\ge 8$ **fals(1pct).**

4. Suma se mai scrie $(10-1)+(10^2-1)+...+(10^{2013}-1)$ **(2pct);**

$=10+10^2+...+10^{2013}-2013$ **(1pct);**

$=10\cdot\frac{10^{2013}-1}{10-1}-2013=\frac{10}{9}(10^{2013}-1)-2013$ **(2pct).**

5. $\cos 160^\circ=-\cos(180^\circ-160^\circ)=-\cos 20^\circ$ **(2pct);** $\cos 140^\circ=-\cos(180^\circ-140^\circ)=-\cos 40^\circ$ **(2pct); suma este zero(1pct).**

6. M **este mijlocul segmentului** BC **deci** $M(3,3)$ **(2pct); ecuaţia medianei din A este** $AM: x-2y+3=0$ **(3pct).**

SUBIECTUL II

1. a) $A^2=\begin{pmatrix}1&4\\0&1\end{pmatrix}$ **(2pct);** $2A=\begin{pmatrix}2&4\\0&2\end{pmatrix}$ **(1pct);** $A^2-2A+I_2=O_2$ **(2pct).**

b) $A^2=\begin{pmatrix}1&4\\0&1\end{pmatrix}$ **(1pct); presupunem** $A^k=\begin{pmatrix}1&2k\\0&1\end{pmatrix}$ **(1pct);**

$A^{k+1}=\begin{pmatrix}1&2k\\0&1\end{pmatrix}\begin{pmatrix}1&2\\0&1\end{pmatrix}=\begin{pmatrix}1&2k+2\\0&1\end{pmatrix}$ **(2pct) deci** $A^n=\begin{pmatrix}1&2n\\0&1\end{pmatrix}$ **(1pct).**

c) $\det A = 1 \neq 0 \Rightarrow \exists A^{-1}$ **(1pct);** $A^t = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ **(1pct);** $A^* = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ **(2pct);**

$A^{-1} = \frac{1}{\det A}\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ **(1pct).**

2 a) verificare prin calcul (5pct)

b) $x \circ x = 19 \Leftrightarrow (x-3)^2 + 3 = 19$ **(1pct);** $(x-3)^2 = 16$ **(1pct);** $x - 3 = \pm 4$ **(1pct);** $x \in \{7, -1\}$ **(2pct)**

c) $x \circ 3 = 3 \circ x = 3, \forall x \in R$ **(2pct);**

$\sqrt[3]{1} \circ \sqrt[3]{2} \circ \sqrt[3]{3} \circ \ldots \circ \sqrt[3]{2013} = (\sqrt[3]{1} \circ \sqrt[3]{2} \circ \sqrt[3]{3} \circ \ldots \circ \sqrt[3]{26}) \circ 3 \circ (\sqrt[3]{28} \circ \ldots \circ \sqrt[3]{2013})$ **(3pct).**
$= 3$

SUBIECTUL III

1. a) $\lim_{x\to\infty} f(x) = +\infty \Rightarrow$ **nu are asimptotă orizontală spre** $+\infty$ **(1pct);**

$m = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^3}{x^3 - x} = 1$ **(1pct);** $n = \lim_{x\to\infty}[f(x) - x] = \lim_{x\to\infty} \frac{x}{x^2 - 1} = 0$ **(2pct);**

Dreapta de ecuaţie $y = x$ **este asimptotă oblică spre** $+\infty$ **(1pct).**

b)
$f'(x) = \frac{3x^2(x^2-1) - x^3 \cdot 2x}{(x^2-1)^2} = \frac{x^4 - 3x^2}{(x^2-1)^2}$ **(1pct);**
$f'(x) = 0 \Rightarrow x_1 = 0, x_2 = 0, x_3 = \sqrt{3}, x_4 = -\sqrt{3}$ **(1pct); pe intervalele** $(-\infty, -\sqrt{3})$ **si** $(\sqrt{3}, +\infty)$ **avem** $f'(x) > 0$ **deci funcţia este strict crescătoare (1pct); pe intervalele** $(-\sqrt{3}, -1)$ **,** $(-1, 0)$**,** $(0, 1)$ **si** $(1, +\sqrt{3})$ **avem** $f'(x) < 0$ **deci funcţia este strict descrescătoare (2pct).**

c) tangenta are ecuaţie de forma $y - f(x_0) = f'(x_0)(x - x_0)$ **(1pct);**

$x_0 = 2, f(x_0) = f(2) = \frac{8}{3}, f'(x_0) = f'(2) = \frac{4}{9}$ **(2pct); deci** $y - \frac{8}{3} = \frac{4}{9}(x-2)$ **(1pct);**

$4x - 9y + 16 = 0$ **(1pct).**

2. a) $f_1(x)=(x-1)e^x$ **(1pct); trebuie ca** $f_1^{'}(x)=f_0(x)$ **(1pct);**

$f_1^{'}(x)=(x-1)^{'}e^x+(x-1)(e^x)^{'}$ **(2pct); =** $e^x+(x-1)e^x=xe^x=f_0(x)$ **(1pct).**

b) $f_0(x)>0$ **pe** $[0,1]$ **(1pct);**

aria $=\int_0^1 f_0(x)dx=\int_0^1 xe^x dx=\int_0^1 x(e^x)^{'}dx=xe^x\Big|_0^1-\int_0^1 (x)^{'}e^x dx=e-\int_0^1 e^x dx$ **(3pct);** $=1$ **(1pct).**

c) $f_{2012}(x^2)=(x^2-2012)e^{x^2}$ **(1pct);** $f_{2013}(x^2)=(x^2-2013)e^{x^2}$ **(1pct);**

$x^2-2012>x^2-2013$ **(1pct);** $f_{2012}(x^2)>f_{2013}(x^2)$ **(1pct);**

$\int_0^1 f_{2012}(x^2)dx\geq\int_0^1 f_{2013}(x^2)dx$ **(1pct).**

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