Inspectoratul Şcolar Judeţean Constanţa

SIMULAREA EXAMENULUI DE BACALAUREAT
MATEMATICĂ -13 MARTIE 2013 -
Proba E.c.
Barem de corectare şi notare

Varianta 1
M_şt-nat

Filiera teoretică, profilul real, specializarea stiinţe ale naturii

SUBIECTUL I

1. $\log_3 9 = 2$ $\qquad \log_{\frac{1}{2}} 4 = -2$ **2p**

$2\log_3^2 9 - 3\log_{\frac{1}{2}} 4 = 2\cdot 4 + 3\cdot 2 = 14$ **3p**

2. $f(1)+\cdots+f(20) = 3(1^2+\cdots+20^2)-(1+\cdots+20)+5(1+\cdots+1)$ ... **3p**

$S = 8500$ **2p**

3. $C_6^4 = 15 \vdots 3$ **1p**

$A_5^2 = 20$ **1p**

$P_4 = 24 \vdots 3$ **1p**

$P = \frac{2}{3}$ **2p**

4. $a_5 = a_1 + 4r$ $(1p) \Rightarrow 4r = 8 \Rightarrow r = 2$ $(\mathbf{1p})$, iar $a_{10} = a_1 + 9r = 3 + 18 = 21$ $(\mathbf{1p})$.

$S_{10} = \frac{(a_1 + a_{10})\cdot 10}{2} = 120$ $(\mathbf{2p})$

5. $\sin 135^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$ **1p**

$\cos 300^0 = \cos 60^0 = \frac{1}{2}$ **1p**

$\cos 240^0 = -\cos 60^0 = -\frac{1}{2}$ **1p**

$\cos 45^0 = \frac{\sqrt{2}}{2}$ **1p**

$\sin 135^0 + \cos 300^0 + \cos 240^0 - \cos 45^0 = 0$ **1p**

6. Teorema cosinusului: $BC^2 = AB^2 + AC^2 - 2AB\cdot AC\cos 120^\circ$ **2p**

$\cos 120^\circ = -\frac{1}{2}$ **2p**

$BC = 7$ **1p**

SUBIECTUL II

1. a) det M(a)= a+1......... **5p**

b) Prin calcul direct **5p**

c) Din b), $M^2(a) = M(a^2+a) = M((a+1)^2-1)$ **1p**

Presupunem că $M^n(a) = M((a+1)^n - 1)$ **1p**

$M^{n+1}(a) = M^n(a)M(a) == M((a+1)^n-1)M(a) = M((a+1)^n a - a + (a+1)^n - 1 + a) = M((a+1)^{n+1}-1)$ **3p**

2. a) $x * y = (x-7)(y-7)+7$ **1p**

$x*x*x = (x-7)^3 + 7$ **2p**

şi deci ecuaţia este $(x-7)^3 + 7 = x$ cu soluţiile $x_1 = 6, x_2 = 7, x_3 = 8$ **2p**

b) Prin calcul direct **5p**

c) Din b), $1*2*\cdots*2013 = (1*\cdots*6)*7*(8*\cdots*2013) = 7$ **5p**

SUBIECTUL III

1. a) $f'(x) = 3x^2 lnx + x^2 = x^2(3lnx + 1)(\mathbf{2p}).\ f(e) + f'(e) = e^3 lne\ (\mathbf{1p}) + e^2(3lne + 1)(\mathbf{1p}) = e^3 + 4e^2 = e^2(e + 4)$ **(1p).**

b) $\lim\limits_{x\to\infty} \frac{f'(x)}{x^2 \ln x} = \lim\limits_{x\to\infty} \frac{x^2(3\ln x + 1)}{x^2 \ln x}(2p) = \lim\limits_{x\to\infty} \frac{3\ln x + 1}{\ln x} = \lim\limits_{x\to\infty}\left(3 + \frac{1}{\ln x}\right)(2p) = 3\,(1p)$

c) $f'(x) = 0 \Rightarrow lnx = -\frac{1}{3} \Rightarrow x = e^{-\frac{1}{3}}$ **(1p)**

$f\left(e^{-\frac{1}{3}}\right) = e^{-1} lne^{-\frac{1}{3}} = -\frac{1}{3e}$ **(1p).**

Folosind monotonia$\Rightarrow$ $f(x) \geq f\left(e^{-\frac{1}{3}}\right)$ **(2p)**, $(\forall)x > 0 \Rightarrow f(x) \geq -\frac{1}{3e}, (\forall)x \in \mathbb{R}$ **(1p)**

2. a) $\int_0^1 f_1(x)\,dx = \int\limits_0^1 \frac{x}{x^2+1}dx = \frac{1}{2}\ln\left(x^2+1\right)\Big|_0^1 = \frac{1}{2}\ln 2$ **5p**

b) $f_{2013}(x) = \frac{x^{2013}}{x^2+1} \Rightarrow f(-x) = -f(x)\ \forall x \in \mathbf{R}$ deci funcţia este impară **3p**

aşadar integrala pe orice interval simetric este zero.. **2p**

c) $f_3(x) = \frac{x^3}{x^2+1}$ **1p**

$f_3'(x) = \frac{x^4 + 3x^2}{\left(x^2+1\right)^2}$ **1p**

$f_3'(x) \geq 0\ \forall x \in [1, 2] \Rightarrow f_3$ crescătoare **1p**

$m = f(1) = \frac{1}{2},\ M = f(2) = \frac{8}{5}$ **1p**

$m(b-a) \leq \int\limits_1^2 f_3(x)dx \leq M(b-a) \Rightarrow \frac{1}{2} \leq \int\limits_1^2 f_3(x)dx \leq \frac{8}{5}$..... **1p**